Warm up

Task 1 : Simplifying a surd $\sqrt{40}$

Task 2 : Addition and subtraction of surds $3\sqrt{5} + 5\sqrt{5}$

Task 3 : Simplifying and adding surds $\sqrt{27} + \sqrt{48}$

Task 4 : Multiplying surds $\sqrt{6} \times \sqrt{18}$

TASK 1

Simplify $\sqrt{180}$

$180 = 2 \times 2 \times 3 \times 3 \times 5$

$= 4 \times 9 \times 5$

$\sqrt{180} = \sqrt{4} \times \sqrt{9} \times \sqrt{5}$

$= 2 \times 3\sqrt{5}$

$= 6\sqrt{5}$

TASK 2

Simplify $3\sqrt{7} + 2\sqrt{7}$

$\sqrt{7} + \sqrt{7} + \sqrt{7} \quad + \quad \sqrt{7} + \sqrt{7}$

$= 5\sqrt{7}$

TASK 3

Simplify $\sqrt{250} + \sqrt{40}$

$\sqrt{250} = \sqrt{25} \times \sqrt{10} = 5\sqrt{10}$

$\sqrt{40} = \sqrt{4} \times \sqrt{10} = 2\sqrt{10}$

$\sqrt{250} + \sqrt{40} = 5\sqrt{10} + 2\sqrt{10}$

$= 7\sqrt{10}$

TASK 4

Simplify $\sqrt{24} \times \sqrt{27}$

$\sqrt{24} = \sqrt{4} \times \sqrt{6} = 2\sqrt{6}$

$\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

$2\sqrt{6} \times 3\sqrt{3} = 6\sqrt{18}$

$= 6 \times \sqrt{9} \times \sqrt{2}$

$= 18\sqrt{2}$

Brackets and Surds

$\sqrt{2}(3+\sqrt{2}) =$?

$(\sqrt{2}+1)(\sqrt{2}-1) =$?

$(\sqrt{8}+3)(\sqrt{2}+5) =$

$=$

$=$

$=$?

$(\sqrt{5}-2)^2 =$

$=$

$=$?

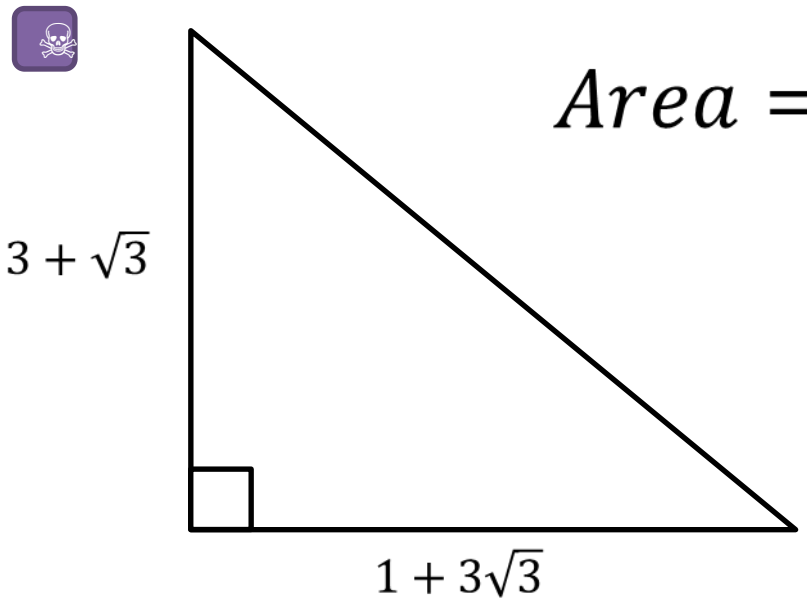
Test Your Understanding

a $\sqrt{5}(2+\sqrt{3}) = ?$

b $(1+\sqrt{3})(2+\sqrt{3}) = ?$

c $(\sqrt{8}-1)(\sqrt{2}+3) = ?$

d $(3-2\sqrt{5})^2 = ?$



$Area = ?$

Practise these specific Key Skills:
https://www.drfrostmaths.com/keyskills.php?permid=125

https://www.drfrostmaths.com/keyskills.php?permid=311

Surds (1)

Simplify

1) $\sqrt{12}$

2) $\sqrt{50}$

3) $\sqrt{72}$

4) $\sqrt{60}$

5) $\sqrt{28}$

6) $\sqrt{96}$

7) $\sqrt{108}$

8) $\sqrt{32}$

ANSWERS

Simplify

1) $\sqrt{12} \times \sqrt{6}$

2) $\sqrt{50} \times \sqrt{8}$

3) $\sqrt{14} \times \sqrt{28}$

4) $\sqrt{30} \times \sqrt{10}$

5) $\sqrt{15} \times \sqrt{45}$

6) $\sqrt{18} \times \sqrt{15}$

7) $\sqrt{120} \times \sqrt{15}$

8) $\sqrt{32} \times \sqrt{8}$

ANSWERS

Simplify

1) $\sqrt{2}(1 + \sqrt{2})$

2) $\sqrt{3}(2 - \sqrt{3})$

3) $\sqrt{3}(2\sqrt{3} + 1)$

4) $\sqrt{2}(3\sqrt{2} - 2)$

5) $2\sqrt{2}(1 + 2\sqrt{2})$

6) $3\sqrt{2}(2 - 2\sqrt{2})$

7) $2\sqrt{5}(3 + 4\sqrt{5})$

8) $6\sqrt{2}(\sqrt{2} - 6)$

ANSWERS

Simplify

1) $\sqrt{12}$ $2\sqrt{3}$

2) $\sqrt{50}$ $5\sqrt{2}$

3) $\sqrt{72}$ $6\sqrt{2}$

4) $\sqrt{60}$ $2\sqrt{15}$

5) $\sqrt{28}$ $2\sqrt{7}$

6) $\sqrt{96}$ $4\sqrt{6}$

7) $\sqrt{108}$ $6\sqrt{3}$

8) $\sqrt{32}$ $4\sqrt{2}$

ANSWERS

Simplify

1) $\sqrt{12} \times \sqrt{6}$ $6\sqrt{2}$

2) $\sqrt{50} \times \sqrt{8}$ 20

3) $\sqrt{14} \times \sqrt{28}$ $14\sqrt{2}$

4) $\sqrt{30} \times \sqrt{10}$ $10\sqrt{3}$

5) $\sqrt{15} \times \sqrt{45}$ $15\sqrt{3}$

6) $\sqrt{18} \times \sqrt{15}$ $3\sqrt{30}$

7) $\sqrt{120} \times \sqrt{15}$ $30\sqrt{2}$

8) $\sqrt{32} \times \sqrt{8}$ 16

ANSWERS

Simplify

1) $\sqrt{2}(1 + \sqrt{2})$ $\sqrt{2} + 2$

2) $\sqrt{3}(2 - \sqrt{3})$ $2\sqrt{3} - 3$

3) $\sqrt{3}(2\sqrt{3} + 1)$ $6 + \sqrt{3}$

4) $\sqrt{2}(3\sqrt{2} - 2)$ $6 - 2\sqrt{2}$

5) $2\sqrt{2}(1 + 2\sqrt{2})$ $2\sqrt{2} + 8$

6) $3\sqrt{2}(2 - 2\sqrt{2})$ $6\sqrt{2} - 12$

7) $2\sqrt{5}(3 + 4\sqrt{5})$ $6\sqrt{5} + 40$

8) $6\sqrt{2}(\sqrt{2} - 6)$ $12 - 36\sqrt{2}$

ANSWERS

Exercise 2

1 Simplify the following:

a $\sqrt{8}+\sqrt{18}=$?

b $\sqrt{12}-\sqrt{3}=$?

c $\sqrt{20}+\sqrt{45}=$?

d $\sqrt{3}+\sqrt{12}+\sqrt{27}=$?

e $\sqrt{300}-\sqrt{48}=$?

f $2\sqrt{50}+3\sqrt{32}=$?

2 Expand and simplify the following, leaving your answers in the form $a+b\sqrt{c}$

a $\sqrt{3}(2+\sqrt{3})=$?

b $(\sqrt{3}+1)(2+\sqrt{3})=$?

c $(\sqrt{5}-1)(2+\sqrt{5})=$?

d $(\sqrt{7}+1)(2-2\sqrt{7})=$?

e $(2-\sqrt{3})^2=$?

3 Expand and simplify:

a $(2-\sqrt{8})(2+\sqrt{2})=$?

b $(3+\sqrt{27})(4-\sqrt{3})=$?

c $(2\sqrt{2}+5)(4+\sqrt{18})=$?

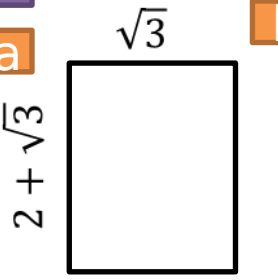
d $(\sqrt{8}+\sqrt{18})(\sqrt{32}-\sqrt{50})=$?

e $(\sqrt{2}+1)^2-(\sqrt{2}-1)^2=$?

f $(\sqrt{3}+2)^2-(\sqrt{3}-2)^2=$?

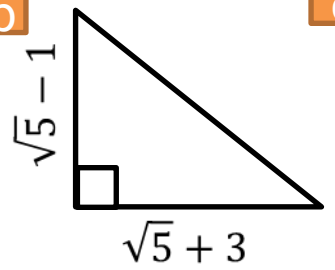
4 Determine the area of :

a



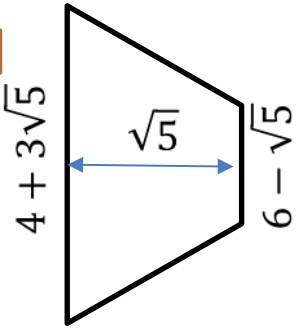
$A=$?

b



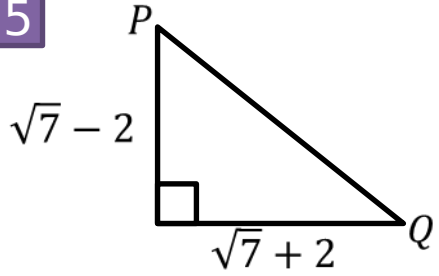
$A=$?

c



$A=$?

5



Find the length of PQ.

?

Rationalising The Denominator

Here's a surd. What could we multiply it by such that it's no longer an irrational number?

$$\sqrt{5} \times \boxed{?} = \boxed{?}$$

$$\frac{1}{\sqrt{2}} \times \boxed{?} = \boxed{?}$$

In this fraction, the denominator is irrational. '**Rationalising the denominator**' means making the denominator a rational number.

What could we multiply this fraction by to both rationalise the denominator, but leave the value of the fraction unchanged?

Fro Side Note: There's two reasons why we might want to do this:

1. For aesthetic reasons, it makes more sense to say "half of root 2" rather than "one root two-th of 1". It's nice to divide by something whole!
2. It makes it easier for us to

$$\frac{\sqrt{7}-1}{\sqrt{3}} \times \bigstar$$

What should replace the star in order to rationalise the denominator?

A

$$\frac{\sqrt{7}+1}{\sqrt{7}+1}$$

B

$$\frac{\sqrt{3}}{\sqrt{3}}$$

C

$$\frac{\sqrt{7}-1}{\sqrt{7}-1}$$

D

$$\sqrt{3}$$

More Examples

$\frac{3}{\sqrt{2}} = ?$

$\frac{6}{\sqrt{3}} = ?$

$\frac{8}{\sqrt{10}} = ?$

$\frac{10}{\sqrt{5}} = ?$

$\frac{7}{\sqrt{7}} = ?$

Test Your Understanding:

a $\frac{12}{\sqrt{3}} = ?$

b $\frac{2}{\sqrt{6}} = ?$

c $\frac{4\sqrt{2}}{\sqrt{8}} = ?$

Practise this specific Key Skill:
https://www.drfrostmaths.com/keyskills.php?permid=126

Rationalise the denominator

$$\frac{5}{\sqrt{5}}$$

Rationalise the denominator

$$\frac{6}{\sqrt{2}}$$

Rationalise the denominator

$$\frac{3}{\sqrt{6}}$$

Rationalise the denominator

$$\frac{12\sqrt{3}}{\sqrt{6}}$$

$$\frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$$

Rationalise the denominator

$$\frac{6}{\sqrt{2}}$$

$$\frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 3\sqrt{2}$$

Rationalise the denominator

$$\frac{3}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{1}{2}\sqrt{6}$$

Rationalise the denominator

$$\frac{12\sqrt{3}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = 6\sqrt{2}$$

More Complex Denominators

You've seen 'rationalising a denominator', the idea being that we don't like to divide things by an irrational number.

But what do we multiply the numerator and denominator by if we have a more complicated denominator?

$$\frac{1}{\sqrt{2}+1} \times \boxed{?} = \boxed{?}$$

We multiply the denominator by what is known as its **conjugate**, i.e. the same expression but the + replaced with $-$ and vice versa. That way, we obtain the difference of two squares. Since $(a+b)(a-b) = a^2 - b^2$, any surds will be squared and thus we'll end up with no surds in the denominator.
And if we've multiplied the denominator by this, we need to multiply the numerator by the same to preserve the value.

More Examples

$$\frac{3}{\sqrt{6}-2} \times \boxed{?} = \boxed{?}$$

You can explicitly expand out $(\sqrt{6}-2)(\sqrt{6}+2)$ in the denominator, but remember that $(a-b)(a+b) = a^2 - b^2$ so we get $6 - 4 = 2$
Just remember: 'difference of two squares'!

$$\frac{4}{\sqrt{3}+1} \times \boxed{?} = \boxed{?} = \boxed{?}$$

Diagnostic Questions

What would be a good first step to rationalise the denominator of:

$$\frac{5}{1+\sqrt{2}}$$

A $\frac{5}{1+\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$

B $\frac{5}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$

C $\frac{5}{1+\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

D $\frac{5}{1+\sqrt{2}} \times \frac{5}{1-\sqrt{2}}$

$$\frac{7}{\sqrt{7}-\sqrt{3}} \times \bigstar$$

What should replace the star in order to rationalise the denominator?

A

$$\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$$

B

$$\frac{\sqrt{4}}{\sqrt{4}}$$

C

$$\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$$

D

$$\sqrt{4}$$

Rationalise the denominator:

$$\frac{1}{6+\sqrt{2}}$$

A

$$\frac{6+\sqrt{2}}{32}$$

B

$$\frac{6-\sqrt{2}}{32}$$

C

$$\frac{6+\sqrt{2}}{34}$$

D

$$\frac{6-\sqrt{2}}{34}$$

$$\frac{3\sqrt{2}+4}{5\sqrt{2}-7} \times \boxed{?} = \boxed{?}$$

$$\frac{\sqrt{2}}{12-\sqrt{11}} = \frac{\square}{\phantom{12-\sqrt{11}}}$$

When you rationalise the denominator, what should replace the rectangle in the final, simplified answer?

A $\sqrt{24}+\sqrt{22}$

B $12\sqrt{2}-\sqrt{22}$

C $\sqrt{22}$

D $12\sqrt{2}+\sqrt{22}$

$$\frac{\sqrt{2}}{12-\sqrt{11}} = \frac{}{\blacksquare}$$

When you rationalise the denominator, what should replace the rectangle in the final, simplified answer?

A

1

B

133

C

23

D

$133-24\sqrt{11}$

Q7.

Which option below is a correct first step to rationalise?

$$\frac{\sqrt{2}+3}{\sqrt{6}-1}$$

A. $\frac{\sqrt{2}+3}{\sqrt{6}-1} \times \frac{\sqrt{2}+3}{\sqrt{6}-1}$

B. $\frac{\sqrt{2}+3}{\sqrt{6}-1} \times \frac{\sqrt{6}}{\sqrt{6}}$

C. $\frac{\sqrt{2}+3}{\sqrt{6}-1} \times \frac{\sqrt{6}-1}{\sqrt{6}-1}$

D. $\frac{\sqrt{2}+3}{\sqrt{6}-1} \times \frac{\sqrt{6}+1}{\sqrt{6}+1}$

Ark

Test Your Understanding

Rationalise the denominator and simplify

$$\frac{4}{\sqrt{5}-2}$$

?

Rationalise the denominator and simplify

$$\frac{2\sqrt{3}-1}{3\sqrt{3}+1}$$

?

Solve $y(\sqrt{3}-1)=8$

Give your answer in the form $a+b\sqrt{3}$ where a and b are integers.

?

Rationalise the denominator

1) $\frac{2}{\sqrt{3}}$

2) $\frac{2\sqrt{5}}{\sqrt{5}}$

3) $\frac{12\sqrt{3}}{\sqrt{6}}$

4) $\frac{8\sqrt{2}}{\sqrt{8}}$

5) $\frac{10\sqrt{8}}{\sqrt{2}}$

6) $\frac{5}{2\sqrt{3}}$

ANSWERS

Rationalise the denominator

1) $\frac{2}{\sqrt{3}+1}$

2) $\frac{\sqrt{5}}{\sqrt{5}-2}$

3) $\frac{3}{\sqrt{2}+1}$

4) $\frac{2\sqrt{2}}{1+\sqrt{2}}$

5) $\frac{4\sqrt{2}}{2-\sqrt{2}}$

6) $\frac{5\sqrt{2}}{2\sqrt{2}-2}$

ANSWERS

Rationalise the denominator

1) $\frac{2+\sqrt{3}}{\sqrt{3}-1}$

2) $\frac{\sqrt{5}-2}{\sqrt{5}+2}$

3) $\frac{3\sqrt{2}+1}{\sqrt{2}-1}$

4) $\frac{2\sqrt{2}-1}{1-\sqrt{2}}$

5) $\frac{2\sqrt{2}+2}{2+\sqrt{2}}$

6) $\frac{\sqrt{3}+1}{2\sqrt{3}-2}$

ANSWERS

Rationalise the denominator

1) $\frac{2}{\sqrt{3}}$

2) $\frac{2\sqrt{5}}{\sqrt{5}}$

3) $\frac{12\sqrt{3}}{\sqrt{6}}$

4) $\frac{8\sqrt{2}}{\sqrt{8}}$

5) $\frac{10\sqrt{8}}{\sqrt{2}}$

6) $\frac{5}{2\sqrt{3}}$

ANSWERS

1) $\frac{2\sqrt{3}}{3}$

2) 2

3) $6\sqrt{2}$

4) 4

5) 20

6) $\frac{5\sqrt{3}}{6}$

Rationalise the denominator

1) $\frac{2}{\sqrt{3}+1}$

2) $\frac{\sqrt{5}}{\sqrt{5}-2}$

3) $\frac{3}{\sqrt{2}+1}$

4) $\frac{2\sqrt{2}}{1+\sqrt{2}}$

5) $\frac{4\sqrt{2}}{2-\sqrt{2}}$

6) $\frac{5\sqrt{2}}{2\sqrt{2}-2}$

ANSWERS

1) $\sqrt{3}-1$

2) $5+2\sqrt{5}$

3) $3\sqrt{2}-3$

4) $4-2\sqrt{2}$

5) $4+4\sqrt{2}$

6) $\frac{10+5\sqrt{2}}{2}$

Rationalise the denominator

1) $\frac{2+\sqrt{3}}{\sqrt{3}-1}$

2) $\frac{\sqrt{5}-2}{\sqrt{5}+2}$

3) $\frac{3\sqrt{2}+1}{\sqrt{2}-1}$

4) $\frac{2\sqrt{2}-1}{1-\sqrt{2}}$

5) $\frac{2\sqrt{2}+2}{2+\sqrt{2}}$

6) $\frac{\sqrt{3}+1}{2\sqrt{3}-2}$

ANSWERS

1) $\frac{5+3\sqrt{3}}{2}$

2) $9-4\sqrt{5}$

3) $7+4\sqrt{2}$

4) $-3-\sqrt{2}$

5) $\sqrt{2}$

6) $2+\sqrt{3}$

Simplifying surds
rationalising the denominator

$\frac{2+\sqrt{2}}{4}$	
$\frac{2+\sqrt{8}}{4}$	
$\frac{2+2\sqrt{8}}{4}$	
$\frac{2(2+\sqrt{200})}{8}$	
$\frac{4+\sqrt{32}}{4}$	
$\frac{(2+\sqrt{2})^2}{4}$	

Write each expression in the form $a + b\sqrt{2}$, where $a, b \in \mathbb{Q}$.

$\frac{1}{2}+\frac{5}{2}\sqrt{2}$

$\sqrt{2}-1$

$2+3\sqrt{2}$

$\frac{1}{2}+\frac{1}{2}\sqrt{2}$

$\frac{1}{2}+\sqrt{2}$

$1+\sqrt{2}$

$\frac{1}{4}\sqrt{2}$

$\sqrt{2}-2$

A final super hard puzzle

Solve $\frac{\sqrt[4]{9}}{\sqrt[5]{27}} = \sqrt[x]{3}$

$$\frac{\sqrt[4]{3^2}}{\sqrt[5]{3^3}} = \frac{\left(3^2\right)^{\frac{1}{4}}}{\left(3^3\right)^{\frac{1}{5}}} = \frac{3^{\frac{1}{2}}}{3^{\frac{3}{5}}} = 3^{-\frac{1}{10}}$$

But $\sqrt[x]{3} = 3^{\frac{1}{x}}$

$$\therefore \frac{1}{x} = -\frac{1}{10} \quad \rightarrow \quad x = -10$$

Exercise 3

1 Rationalise the denominator and simplify the following:

a $\dfrac{1}{\sqrt{5}+2} =$?

b $\dfrac{\sqrt{3}}{\sqrt{3}-1} =$?

c $\dfrac{\sqrt{5}+1}{\sqrt{5}-2} =$?

d $\dfrac{2\sqrt{3}-1}{3\sqrt{3}+4} =$?

e $\dfrac{5\sqrt{5}-2}{2\sqrt{5}-3} =$?

2 Expand and simplify:

$(\sqrt{5}+3)(\sqrt{5}-2)(\sqrt{5}+1) =$?

3 Rationalise the denominator, giving your answer in the form $a+b\sqrt{3}$.

$\dfrac{3\sqrt{3}+7}{3\sqrt{3}-5} =$?

4 Solve $x(4-\sqrt{6}) = 10$ giving your answer in the form $a+b\sqrt{6}$.

?

5 Solve $y(1+\sqrt{2}) - \sqrt{2} = 3$

$y = \dfrac{3+\sqrt{2}}{1+\sqrt{2}} =$?

6 Simplify:

$\dfrac{\sqrt{a+1}-\sqrt{a}}{\sqrt{a+1}+\sqrt{a}} =$?

Friday's test

Extra time students?

Periods 2 & 3 - Paper A - Non-calculator (50 marks)

Period 7 - Paper B - Ca[illegible]

St. Louis School
CARPE MAGNIFICENTIAM

Assessment Test 4a
Year 10

Name _____ TIME ALLOWED: 60 Minutes

Maths Teacher (circle **ONE**)

Mr Ellison Miss Erskine Mr Guite Miss Koppinen

Miss Cozzi Mr Shaw Mr Spencer

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- Calculators must **not** be used in this paper.
- You may use tracing paper.
- You must show all necessary working clearly and you will be given marks for correct methods even if your answer is incorrect.
- All answers should be given in their simplest form.

The total marks for this test is [50]

St. Louis School
CARPE MAGNIFICENTIAM

Assessment Test 4b
Year 10

Name _____ TIME ALLOWED: 35 Minutes

Maths Teacher (circle **ONE**)

Mr Ellison Miss Erskine Mr Guite Miss Koppinen

Miss Cozzi Mr Shaw Mr Spencer

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly and you will be given marks for correct methods, including sketches, even if your answer is incorrect.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- For π, use your calculator value.

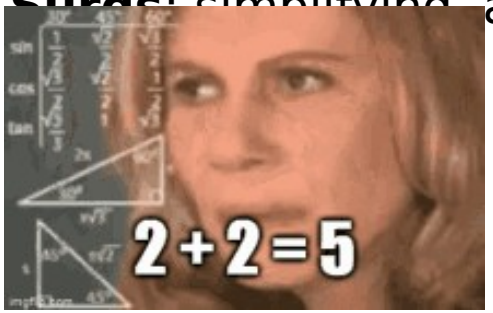
The total marks for this test is [29]

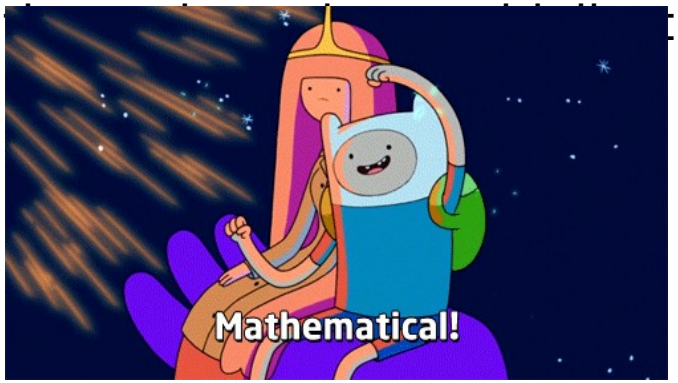
Raw Score	%	Target Grade (if applicable)

Friday’s test - topics

Generally: Everything you’ve covered in IGCSE Maths

- **Number** - e.g. percentages, highest common factor, powers and roots, ratio and proportion, standard form, meaning and rules of exponents, estimating & rounding, converting, problems with speed/time/distance
- **Algebra**: linear equations, simplification of algebraic functions, derivation and rearrangement of formulae
- **Cartesian plane**: co-ordinates, distance, mid-points, gradient, equation of a straight line
- **Powers:** standard form, rules for exponents, multiplying and dividing indices, negative indices
- **Percentages**: simple interest, compound interest, percentiles
- **Sequences**: nth term, difference method, linear, quadratic, cubic & geometric sequences
- **Shapes**: area, perimeter, arc length, sectors, surface area, volume
- **Surds**: simplifying, addi tion, rationalising the





I promise we’ll do a Kahoot next

Formula List

For the equation $ax^2 + bx + c = 0$ $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Curved surface area, A, of cylinder of radius r, height h. $\quad A = 2\pi rh$

Curved surface area, A, of cone of radius r, sloping edge l. $\quad A = \pi rl$

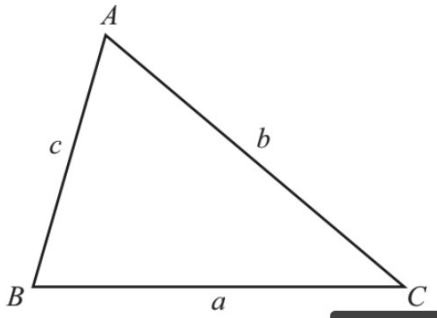
Curved surface area, A, of sphere of radius r. $\quad A = 4\pi r^2$

Volume, V, of pyramid, base area A, height h. $\quad V = \dfrac{1}{3}Ah$

Volume, V, of cylinder of radius r, height h. $\quad V = \pi r^2 h$

Volume, V, of cone of radius r, height h. $\quad V = \dfrac{1}{3}\pi r^2 h$

Volume, V, of sphere of radius r. $\quad V = \dfrac{4}{3}\pi r^3$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area} = \frac{1}{2}bc\sin A$$